Exercise sheet 5

Algebraic Topology

November 14, 2023

Exercise 1. Let X be a CW complex and let C be a compact subset. Show that C only intersects a finite number of cells.

Exercise 2. Consider T^2 and the circles A and B defined in the proof of Theorem 2.70. Show that $T^2/(A \cup B)$ is homeomorphic to S^2.

Exercise 3. Let $K_n = \{x \in R^2 \colon \|x - (n^{-1}, 0)\| = n^{-1}\}$ be the circle with radius $\frac{1}{n}$ around the point $\left(\frac{1}{n}, 0\right)$ in $\mathbb{R}^2$. As in example 3.7, we let the **Hawaiian earring** $H = \bigcup_{n \in \mathbb{N}} K_n$ be the union of the circles K_n and we equip H with the induced topology of $\mathbb{R}^2$. Show that H cannot be viewed as a CW complex.

Exercise 4. From lasts weeks exercise sheet, assignments 1 - 3 can be solved using Mayer-Vietoris or using Excision. Do these exercises again using the technique you haven't used before.