Assignment 11

MATH-F310: Differential Geometry I

December 19, 2022

1. Show that the tangent bundle to S^1 is diffeomorphic to the cylinder $S^1 \times \mathbb{R}$.
2. A smooth vector field v on a smooth manifold X is said to be complete, if for each $p \in X$, the maximal integral curve of v through p, has domain equal to $\mathbb{R}$. Determine which of the following vector fields are complete:

$$(a) v(x, y) = (1, 0), X = \mathbb{R}^2$$
$$(b) v(x, y) = (1, 0), X = \mathbb{R}^2 \setminus \{(0, 0)\}$$
$$(c) v(x, y) = (-y, x), X = \mathbb{R}^2 \setminus \{(0, 0)\}$$
$$(d) v(x, y) = (1 + x^2, 0), X = \mathbb{R}^2$$

3. A point $x \in X$, is a zero of the vector field v if $v(x) = 0$. Show that if k is odd, there exists a vector field v on S^k having no zeros. It is a rather deep topological fact that nowhere vanishing vector fields do not exist on the even spheres.
4. Prove that if S^k has a nowhere vanishing vector field, then its antipodal map is homotopic to the identity.
5. Prove that the set of all 2×2 matrices of rank 1 is a three-dimensional submanifold of $\mathbb{R}^4 = M(2)$. [Hint: Use the determinant function det:$M(2) \setminus \{0\} \to \mathbb{R}$]