Assignment 5

MATH-F310: Differential Geometry I

October 27, 2023

1. Show that if S is a connected surface in $\mathbb{R}^3$ and $g : S \to \mathbb{R}$ is smooth and takes on only the values $+1$ and -1, then g is constant. Show by example that if S is not connected, then the result fails.
2. The holomorphic map $z \mapsto z^2$ extends to a smooth map $f : S^2 \to S^2$ using stereographic projection. Compute the diferential of this map at a point $p \in S^2$ and find out the critical points of f.
3. Prove that the vector space of smooth functions on a surface is infinite dimensional.
4. Show that the two orientations of the sphere $x_1^2 + x_2^2 + x_3^2 = r^2$ of radius r, is determined by the two normal fields $N_1(p) = (p, p/r)$ and $N_2(p) = (p, -p/r)$.
5. Let S be a surface on $\mathbb{R}^3$ and let $p_0 \in \mathbb{R}^3 \setminus S$. Show that the shortest line segment from p_0 to S (if one exists) is perpendicular to S, i.e., show that if $p \in S$ such that $||p_0 - p||^2 \leq ||p_0 - q||^2$ for all $q \in S$, then the line segment $p_0 - p$ at p is perpendicular to T_pS.