Assignment 7

MATH-F310: Differential Geometry I

November 24, 2022

1. Compute the Gauss curvature of the hyperboloid $x^2 + y^2 - z^2 = 1$ at the point $(1, 0, 0)$.
2. Compute the Gauss map, the shape operator and the Gauss curvature for the cylinder $\{(x, y, z) \mid x^2 + y^2 = r^2\} \subset \mathbb{R}^3$.
3. Let S be a compact connected oriented surface in $\mathbb{R}^3$. Prove that the Gauss map maps S onto the unit sphere S^2.
4. Show that an orientable connected surface whose Gauss and mean curvatures are identically zero must be an open subset of a plane.
5. Let $f = f(x, y)$ be a smooth function on $\mathbb{R}^2$, and let $S \subset \mathbb{R}^3$ be its graph. Suppose that

$$f(0) = \frac{\partial f}{\partial x}(0) = \frac{\partial f}{\partial y}(0) = 0$$

We denote the Hessian matrix of f by: $\begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix}$. Show that the Gauss curvature of S at $(0, 0, 0)$ is $\frac{1}{4}$ times the determinant of the Hessian matrix of f at the origin.